# Performance Comparison of Energy-Efficient Power Control for CDMA and Multiuser UWB Networks

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Abstract—This paper studies the performance of a wireless data network using energy-efficient power control techniques when different multiple access schemes, namely direct-sequence code division multiple access (DS-CDMA) and impulse-radio ultrawideband (IR-UWB), are considered. Due to the large bandwidth of the system, the multipath channel is assumed to be frequency-selective. By making use of noncooperative game-theoretic models and large-system analysis tools, explicit expressions for the achieved utilities at the Nash equilibrium are derived in terms of the network parameters. A measure of the loss of DS-CDMA with respect to IR-UWB is proposed, which proves substantial equivalence between the two schemes. Simulation results are provided to validate the analysis.

# I. INTRODUCTION

The increasing demand for high-speed data services in wireless networks calls for multiple access schemes with efficient resource allocation and interference mitigation. Both direct-sequence code division multiple access (DS-CDMA) and impulse-radio ultrawideband (IR-UWB) are considered to be potential candidates for such next-generation high-speed networks. Design of reliable systems must include transmitter power control, which aims to allow each mobile terminal to achieve the required quality of service at the uplink receiver while minimizing power consumption. Scalable techniques for energy-efficient power control can be derived using game theory [1]–[3].

This paper compares the performance of game-theoretic power control schemes in the uplink of an infrastructure network using either DS-CDMA or IR-UWB as a multiple access technique. The performance index here is represented by the achieved utility at the Nash equilibrium, where utility is defined as the ratio of the throughput to the transmit power. Due to the large bandwidth occupancy [4], the channel fading is assumed to be frequency-selective. Resorting to a large-system analysis [3], systems with equal spreading factor operating in a dense multipath environment are compared. Both analytical and numerical results show that, even though UWB-based networks always outperform CDMA-based systems, the difference between achieved utilities is so slight that equivalence in terms of energy efficiency can be assumed.

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The remainder of the paper is organized as follows. The system model is described in Sect. II. Sect. III contains the main results of the proposed noncooperative power control game. A comparison between the energy efficiency of the two considered multiple access schemes is performed in Sect. IV, where also simulation results are shown. Some conclusions are drawn in Sect. V.

### II. SYSTEM MODEL

# A. IR-UWB Wireless Networks

We consider the uplink of a binary phase shift keying (BPSK) random time-hopping (TH) IR-UWB system with K users transmitting to a common concentration point. The transmitted signal from user k is [5]

$$s_{tx}^{(k)}(t) = \sqrt{\frac{p_k T_f}{N}} \sum_{n=-\infty}^{+\infty} d_n^{(k)} b_{\lfloor n/N_f \rfloor}^{(k)} w_{tx}(t - nT_f - c_n^{(k)} T_c),$$
(1)

where  $w_{tx}(t)$  is the transmitted UWB pulse with duration  $T_c$  and unit energy;  $p_k$  is the transmit power of user k;  $T_f$  is the frame time;  $b_{\lfloor n/N_f \rfloor}^{(k)} \in \{-1,+1\}$  is the information symbol transmitted by user k; and  $N = N_f \cdot N_c$  is the processing gain, where  $N_f$  is the number of pulses representing one information symbol, and  $N_c = T_f/T_c$  denotes the number of possible pulse positions in a frame. Throughout this analysis, a system with polarity code randomization is considered [6]. In particular, the polarity code for user k is  $\mathbf{d}_k = \{d_0^{(k)}, \cdots, d_{N_f-1}^{(k)}\}$ , where the  $d_j^{(k)}$ 's are independent random variables taking values  $\pm 1$  with probability 1/2. To allow the channel to be shared by many users without causing catastrophic collisions, a TH sequence  $\mathbf{c}_k = \{c_1^{(k)}, \cdots, c_{N_f}^{(k)}\}$  is assigned to each user, where  $c_n^{(k)} \in \{0, 1, \cdots, N_c - 1\}$  with equal probability  $1/N_c$ .

Defining a sequence  $\{s_n^{(k)}\}$  as

$$s_n^{(k)} = \begin{cases} d_{\lfloor n/N_c \rfloor}^{(k)}, & c_{\lfloor n/N_c \rfloor \cdot N_c}^{(k)} = n - \lfloor n/N_c \rfloor \cdot N_c, \\ 0, & \text{otherwise,} \end{cases}$$
 (2)

we can express (1) as

$$s_{tx}^{(k)}(t) = \sqrt{\frac{p_k T_f}{N}} \sum_{n=1}^{+\infty} s_n^{(k)} b_{\lfloor n/N \rfloor}^{(k)} w_{tx}(t - nT_c).$$
 (3)

It is worth noting that this system makes use of a ternary sequence  $\{-1,0,+1\}$ , where also the elements are dependent, due to the TH sequence.

The transmission is assumed to be over *frequency-selective* channels, with the channel for user k modeled as a tapped delay line:

$$c_k(t) = \sum_{l=1}^{L} \alpha_l^{(k)} \delta(t - (l-1)T_c - \tau_k), \tag{4}$$

where L is the number of channel paths;  $\alpha_k = [\alpha_1^{(k)}, \ldots, \alpha_L^{(k)}]^T$  and  $\tau_k$  are the fading coefficients and the delay of user k, respectively. Considering a chip-synchronous scenario, symbols are misaligned by an integer multiple of  $T_c$ :  $\tau_k = \Delta_k T_c$ , for every k, where  $\Delta_k$  is uniformly distributed in  $\{0,1,\ldots,N-1\}$ . We also assume that channel characteristics remain unchanged over several symbol intervals [5].

Due to the high resolution of UWB signals, multipath channels can have hundreds of multipath components, especially in indoor environments. To mitigate the effect of multipath fading as much as possible, we consider an access point where K Rake receivers [7] are used. The Rake receiver for user k is in general composed of L coefficients, where the vector  $\boldsymbol{\beta}_k = \mathbf{G} \cdot \boldsymbol{\alpha}_k = [\beta_1^{(k)}, \dots, \beta_L^{(k)}]^T$  represents the combining weights for user k, and the  $L \times L$  matrix  $\mathbf{G}$  depends on the type of Rake receiver employed.

The signal-to-interference-plus-noise ratio (SINR) of the kth user at the output of the Rake receiver can be well approximated (for large  $N_f$ , typically at least 5) by [5]

$$\gamma_{k} = \frac{h_{k}^{(SP)} p_{k}}{h_{k}^{(SI)} p_{k} + \sum_{\substack{j=1\\ j \neq k}}^{K} h_{kj}^{(MAI)} p_{j} + \sigma^{2}},$$
(5)

where  $\sigma^2$  is the variance of the additive white Gaussian noise (AWGN); and the terms due to signal part (SP), self-interference (SI), and multiple access interference (MAI), are

$$h_k^{(\mathrm{SP})} = \boldsymbol{\beta}_k^H \cdot \boldsymbol{\alpha}_k, \tag{6}$$

$$h_k^{(\text{SI})} = \frac{1}{N} \frac{\left| \left| \mathbf{\Phi} \cdot \left( \mathbf{B}_k^H \cdot \boldsymbol{\alpha}_k + \mathbf{A}_k^H \cdot \boldsymbol{\beta}_k \right) \right| \right|^2}{\boldsymbol{\beta}_k^H \cdot \boldsymbol{\alpha}_k}, \tag{7}$$

$$h_{kj}^{(\text{MAI})} = \frac{1}{N} \frac{\left| \left| \mathbf{B}_{k}^{H} \cdot \boldsymbol{\alpha}_{j} \right| \right|^{2} + \left| \left| \mathbf{A}_{j}^{H} \cdot \boldsymbol{\beta}_{k} \right| \right|^{2} + \left| \boldsymbol{\beta}_{k}^{H} \cdot \boldsymbol{\alpha}_{j} \right|^{2}}{\boldsymbol{\beta}_{k}^{H} \cdot \boldsymbol{\alpha}_{k}},$$
(8)

respectively, where the matrices

$$\mathbf{A}_{k} = \begin{pmatrix} \alpha_{L}^{(k)} & \cdots & \cdots & \alpha_{2}^{(k)} \\ 0 & \alpha_{L}^{(k)} & \cdots & \alpha_{3}^{(k)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \alpha_{L}^{(k)} \\ 0 & \cdots & \cdots & 0 \end{pmatrix}, \tag{9}$$

<sup>1</sup>For ease of calculation, perfect channel estimation is considered throughout the paper.

$$\mathbf{B}_{k} = \begin{pmatrix} \beta_{L}^{(k)} & \cdots & \cdots & \beta_{2}^{(k)} \\ 0 & \beta_{L}^{(k)} & \cdots & \beta_{3}^{(k)} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \beta_{L}^{(k)} \\ 0 & \cdots & \cdots & 0 \end{pmatrix},$$
(10)

$$\mathbf{\Phi} = \operatorname{diag}\left\{\phi_1, \dots, \phi_{L-1}\right\}$$
, and  $\phi_l = \sqrt{\frac{\min\{L-l, N_c\}}{N_c}}$ , (11)

have been introduced for convenience of notation.

# B. DS-CDMA Wireless networks

In order to perform a fair comparison, the uplink of a random DS-CDMA system with spreading factor N and K users is considered. It can be noticed that (3) can represent a DS-CDMA system with processing gain N by considering the special case when  $T_f = T_c$  (and thus  $N_c = 1$ ) [5]. As is apparent from (2), using  $N_c = 1$  yields the elements of  $\{s_n^{(k)}\}$  to be binary independent and identical distributed (i.i.d.).

Hence, in a dense frequency-selective multipath environment, the SINR of user k at the output of the Rake receiver is also represented by (5), under the conditions  $N_c = 1$ ,  $N = N_f$ .

# III. THE NONCOOPERATIVE POWER CONTROL GAME

Consider now the application of noncooperative power control techniques to the wireless networks described above. Focusing on mobile terminals, where it is often more important to maximize the number of bits transmitted per Joule of energy consumed than to maximize throughput, a game-theoretic energy-efficient approach like the one described in [3] can be considered.

# A. Analysis of the Nash equilibrium

It is possible to formulate a noncooperative power control game in which each user seeks to maximize its own utility function. Let  $G = [\mathcal{K}, \{P_k\}, \{u_k(\mathbf{p})\}]$  be the proposed noncooperative game where  $\mathcal{K} = \{1, \ldots, K\}$  is the index set for the users;  $P_k = [\underline{p}_k, \overline{p}_k]$  is the strategy set, with  $\underline{p}_k$  and  $\overline{p}_k$  denoting minimum and maximum power constraints, respectively; and  $u_k(\mathbf{p})$  is the payoff function for user k [2], defined as

$$u_k(\mathbf{p}) = \frac{D}{M} R_k \frac{f(\gamma_k)}{p_k},\tag{12}$$

where  $\mathbf{p}=[p_1,\ldots,p_K]$  is the vector of transmit powers; D and M are the number of information bits and the total number of bits in a packet, respectively;  $R_k$  and  $\gamma_k$  are the transmission rate and the SINR for the kth user, respectively; and  $f\left(\gamma_k\right)$  is the efficiency function representing the packet success rate (PSR), i.e., the probability that a packet is received without an error. Throughout this analysis, we assume  $\underline{p}_k=0$  and  $\overline{p}_k=\overline{p}$  for all  $k\in\mathcal{K}$ .

Provided that the efficiency function is increasing, S-shaped, and continuously differentiable, with f(0) = 0,  $f(+\infty) = 1$ , and  $f'(0) = df(\gamma_k)/d\gamma_k|_{\gamma_k=0} = 0$ , the solution of the

$$\nu\left(\Lambda,r,\rho\right) = \begin{cases} \frac{\Lambda(\Lambda^{\rho}-1)\left(4\Lambda^{2r}+3\Lambda^{\rho}-1\right)-2\Lambda^{r+\rho}(\Lambda^{r}+3\Lambda-1)\rho\log\Lambda}{2(\Lambda^{r}-1)^{2}\rho\Lambda^{1+\rho}\log\Lambda}, & \text{if } 0 \leq \rho \leq \min(r,1-r); & (19a) \\ \frac{\Lambda(4\Lambda^{\rho}-1)\left(\Lambda^{2r}-1\right)-2\Lambda^{r+\rho}(3\Lambda r-\rho+\Lambda^{r}\rho)\log\Lambda}{2(\Lambda^{r}-1)^{2}\rho\Lambda^{1+\rho}\log\Lambda}, & \text{if } r \leq \rho \leq 1-r \text{ and } \\ \frac{r \leq 1/2; & (19b)}{2(\Lambda^{r}-1)^{2}\rho\Lambda^{2+\rho}\log\Lambda}, & \text{if } 1-r \leq \rho \leq r \text{ and } \\ \frac{-4\Lambda^{2+2r}-4\Lambda^{2+\rho}+\Lambda^{2(r+\rho)}+4\Lambda^{2+2r+\rho}+3\Lambda^{2+2\rho}-2\Lambda^{1+r+\rho}(r+3\Lambda\rho+\Lambda^{r}\rho-1)\log\Lambda}{2(\Lambda^{r}-1)^{2}\rho\Lambda^{2+\rho}\log\Lambda}, & \text{if } 1-r \leq \rho \leq r \text{ and } \\ \frac{-\Lambda^{2+2r}-4\Lambda^{2+\rho}+\Lambda^{2(r+\rho)}+4\Lambda^{2+2r+\rho}-2\Lambda^{1+r+\rho}(r+3\Lambda\rho+\Lambda^{r}\rho-1)\log\Lambda}{2(\Lambda^{r}-1)^{2}\rho\Lambda^{2+\rho}\log\Lambda}, & \text{if } \max(r,1-r) \leq \rho \leq 1; & (19d) \\ \frac{-\Lambda^{2+2r}-4\Lambda^{2+\rho}+\Lambda^{2(r+\rho)}+4\Lambda^{2+2r+\rho}-2\Lambda^{1+r+\rho}(r+3\Lambda\rho+\Lambda^{r}\rho-1)\log\Lambda}{2(\Lambda^{r}-1)^{2}\rho\Lambda^{2+\rho}\log\Lambda}, & \text{if } \rho \geq 1. & (19e) \end{cases}$$

maximization problem  $\max_{p_k \in P_k} u_k(\mathbf{p})$  for  $k = 1, \dots, K$  is [3]

$$p_k^* = \min \left\{ \frac{\gamma_k^* \left( \sum_{j \neq k} h_{kj}^{(\text{MAI})} p_j + \sigma^2 \right)}{h_k^{(\text{SP})} \left( 1 - \gamma_k^* / \gamma_{0,k} \right)}, \overline{p} \right\}, \quad (13)$$

where

$$\gamma_{0,k} = \frac{h_k^{(\text{SP})}}{h_k^{(\text{SI})}} = N \cdot \frac{(\boldsymbol{\beta}_k^H \cdot \boldsymbol{\alpha}_k)^2}{\left|\left|\boldsymbol{\Phi} \cdot \left(\mathbf{B}_k^H \cdot \boldsymbol{\alpha}_k + \mathbf{A}_k^H \cdot \boldsymbol{\beta}_k\right)\right|\right|^2} \ge 1 \tag{14}$$

and  $\gamma_k^*$  is the solution of

$$f'(\gamma_k^*)\gamma_k^* (1 - \gamma_k^*/\gamma_{0,k}) = f(\gamma_k^*),$$
 (15)

where  $f'(\gamma_k^*) = df(\gamma_k)/d\gamma_k|_{\gamma_k = \gamma_k^*}$ . Since  $\gamma_k^*$  depends only on  $\gamma_{0,k}$ , for convenience of notation a function  $\Gamma(\cdot)$  is defined such that  $\gamma_k^* = \Gamma(\gamma_{0,k})$  [3].

# B. Large-System Analysis

As can be verified in (13), the amount of transmit power  $p_k^*$  required to achieve the target SINR  $\gamma_k^*$  will depend not only on  $h_k^{(\mathrm{SP})}$ , but also on the SI term  $h_k^{(\mathrm{SI})}$  (through  $\gamma_{0,k}$ ) and the MAI (through  $h_{kj}^{(\mathrm{MAI})}$ ). To derive quantitative results for the transmit powers independent of SI and MAI terms, it is possible to resort to the large-system analysis described in [3].

For ease of calculation, the expressions derived in the remainder of the paper consider the following assumptions:

- The channel gains are assumed to be independent complex Gaussian random variables with zero means and variances  $\sigma_{k_l}^2$ , i.e.,  $\alpha_k^{(l)} \sim \mathcal{CN}(0, \sigma_{k_l}^2)$ . This assumption leads  $|\alpha_k^{(l)}|$  to be Rayleigh-distributed with parameter  $\sigma_{k_l}^2/2$ . Although channel modeling for wideband systems is still an open issue, this hypothesis, appealing for its analytical tractability, also provides a good approximation for multipath propagation in UWB systems [8].
- The average power delay profile (aPDP) [9] is assumed to decay exponentially, as is customary used in many UWB channel models [4]. This translates into the hypothesis  $\sigma_{k_l}^2 = \sigma_k^2 \cdot \Lambda^{-\frac{l-1}{L-1}},$  where  $\Lambda = \sigma_{k_1}^2/\sigma_{k_L}^2$  and  $\sigma_k^2$  depends on the distance between user k and the access point.
- Partial-Rake (PRake) receivers with  $L_P$  fingers using maximal ratio combining (MRC) are implemented at the

access point. In other words, we consider G to be a deterministic diagonal matrix, with

$$\{\mathbf{G}\}_{ll} = \begin{cases} 1, & 1 \le l \le r \cdot L, \\ 0, & \text{elsewhere,} \end{cases}$$
 (16)

where  $r \triangleq L_P/L$  and  $0 < r \le 1$ . It is worth noting that, when r = 1, an all-Rake (ARake) is implemented.

- As is typical in multiuser wideband systems, the number
  of users is much smaller than the processing gain, i.e.,
  N >> K. This assumption can also be justified since the
  analysis is performed for dense multipath environments,
  as shown in the following.
- The maximum transmit power  $\overline{p}$  is assumed to be sufficiently large.

Under the above hypotheses, a large-system analysis can be performed considering a dense multipath environment, with  $L \to \infty$ . It turns out that the achieved utilities  $u_k^*$  at the Nash equilibrium converge almost surely (a.s.) to [3], [10]

$$u_{k}^{*} \stackrel{a.s.}{\to} h_{k}^{(SP)} \cdot \frac{D}{M} R_{k} \frac{f\left(\Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right)\right)}{\sigma^{2} \Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right)} \times \left(1 - \frac{\Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) \left[(K - 1)\mu\left(\Lambda, r\right) + \nu\left(\Lambda, r, \rho\right)\right]}{N}\right), \tag{17}$$

where  $r \triangleq L_P/L$ , with  $0 < r \le 1$ ;  $\rho \triangleq N_c/L$ , with  $0 < \rho < \infty$ ;

$$\mu\left(\Lambda, r\right) = \frac{(\Lambda - 1) \cdot \Lambda^{r-1}}{\Lambda^r - 1};\tag{18}$$

and  $\nu\left(\Lambda,r,\rho\right)$  is defined as in (19a)-(19e), shown at the top of the page.

## IV. PERFORMANCE COMPARISON

# A. Analytical Results

The results derived in the previous section allow the performance of IR-UWB and DS-CDMA systems to be compared in terms of achieved utilities at the Nash equilibrium.

For an IR-UWB system, the utility  $u_{k_U}^*$  can be evaluated using (17). In the case of a DS-CDMA system, (17) can still

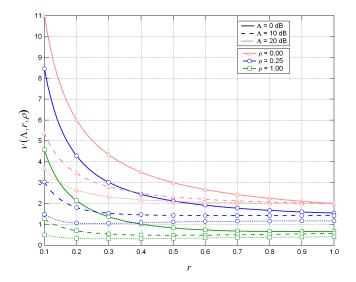


Fig. 1. Shape of  $\nu$   $(\Lambda, r, \rho)$  versus r for some values of  $\Lambda$  and  $\rho$ .

give the utility  $u_{k_C}^*$ , provided that  $\nu\left(\Lambda,r,\rho\right)$  is replaced with  $\nu_0\left(\Lambda,r\right)$ , defined as

$$\nu_0\left(\Lambda, r\right) = \lim_{\rho \to 0} \nu\left(\Lambda, r, \rho\right) = \frac{\Lambda + \Lambda^r - 2\Lambda^{1+r}}{\Lambda - \Lambda^{1+r}}.$$
 (20)

This results is obtained letting  $\rho$  go to 0 in (19a). The proof, omitted because of space limitation, can be derived using the theorems presented in [10] with  $N_c=1$ .

Fig. 1 shows the shape of  $\nu\left(\Lambda,r,\rho\right)$  as a function of r for some values of  $\Lambda$  and  $\rho$ . With a slight abuse of notation,  $\nu_0\left(\Lambda,r\right)$  is reported as  $\nu\left(\Lambda,r,0\right)$  (triangular markers), while circles and square markers depict  $\rho=0.25$  and  $\rho=1.0$ , respectively. As can be noted,  $\nu_0\left(\Lambda,r\right)>\nu\left(\Lambda,r,\rho_1\right)>\nu\left(\Lambda,r,\rho_2\right)$  for any  $\rho_2>\rho_1>0$ . This result is justified by the higher resistance to multipath due to increasing the length of a single frame [3], [5]. Furthermore, keeping  $\rho$  fixed,  $\nu\left(\Lambda,r,\rho\right)$  decreases both as  $\Lambda$  and as r increases. The first behavior makes sense, since the effect of multipath (and thus of SI) is higher in channels with lower  $\Lambda$ . The second behavior reflects the fact that exploiting the diversity by adding a higher number of fingers (and thus increasing r) results in better mitigating the frequency-selective fading.

Proposition 1: When  $L\to\infty$ , the loss  $\Phi$  of a CDMA system with respect to (wrt) an IR-UWB scheme with  $N_c$  possible pulse positions converges a.s. to

$$\Phi \triangleq 10 \log_{10}(u_{k_{II}}^*/u_{k_C}^*) \stackrel{a.s.}{\rightarrow} (10 \log_{10} e) \cdot \varphi \qquad [dB] \quad (21)$$

where

$$\varphi \triangleq \frac{\Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) \cdot \Delta\nu\left(\Lambda, r, \rho\right)}{N - \Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) \cdot \left[\left(K - 1\right)\mu\left(\Lambda, r\right) + \nu\left(\Lambda, r, \rho\right)\right]},$$
(22)

with  $\Delta \nu (\Lambda, r, \rho) = \nu_0 (\Lambda, r) - \nu (\Lambda, r, \rho)$ .

*Proof:* Recalling (15), it can be noted that the slope of  $\Gamma(\gamma_{0,k})$  is very small for large values of  $\gamma_{0,k}$ . Using

TABLE I LIST OF PARAMETERS USED IN THE SIMULATIONS.

M, total number of bits per packet	100 b
D, number of information bits per packet	100 b
$R_k = R$ , bit rate	100 kb/s
$\sigma^2$ , AWGN power at the receiver	$5 \times 10^{-16} \mathrm{W}$
$\overline{p}$ , maximum power constraint	$1\mu\mathrm{W}$

the hypothesis  $N\gg K>1$ , a good approximation for  $\Gamma\left(N/\nu_0\left(\Lambda,r\right)\right)$  is  $\Gamma\left(N/\nu\left(\Lambda,r,\rho\right)\right)$ . Therefore, using (17),

$$\frac{u_{k_{U}}^{*}}{u_{k_{C}}^{*}} \approx \frac{N - \Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) \cdot \left[(K - 1)\mu\left(\Lambda, r\right) + \nu\left(\Lambda, r, \rho\right)\right]}{N - \Gamma\left(\frac{N}{\nu(\Lambda, r, \rho)}\right) \cdot \left[(K - 1)\mu\left(\Lambda, r\right) + \nu_{0}\left(\Lambda, r\right)\right]}$$

$$= \frac{1}{1 - \omega}, \tag{24}$$

with  $\varphi$  defined as in (22). Recalling that  $N \gg 1$ , it is easy to verify that  $\varphi \ll 1$ . Hence, using a first-order Taylor series approximation, the result (21) is straightforward.

As already specified (see also Fig. 1),  $\Delta \nu (\Lambda, r, \rho) > 0$  for any  $\rho > 0$ . Proposition 1 thus states that, using an equal spreading factor in the same multipath scenario, any UWB system outperforms the corresponding CDMA schemes.

Nevertheless, typical values of the network parameters yield very small values of  $\Phi$ , especially as N increases.<sup>2</sup> Hence, using game-theoretic power control techniques, performance of the two multiple access schemes is practically equivalent.

The validity of these claims is verified in the next subsection using numerical simulations.

# B. Numerical Results

Simulations are performed using the iterative algorithm described in detail in [3]. The systems we examine have the design parameters listed in Table I. We use the efficiency function  $f(\gamma_k) = (1 - \mathrm{e}^{-\gamma_k/2})^M$  as a reasonable approximation to the PSR [2], [5]. To model the UWB scenario, the channel gains are assumed as in Sect. III, with  $\sigma_k^2 = 0.3 d_k^{-2}$ , where  $d_k$  is the distance between user k and the access point. Distances are assumed to be uniformly distributed between 3 and 30 m.

Fig. 2 shows a comparison between analytical and simulated normalized utilities  $u_k^*/h_k^{\rm (SP)}$  at the Nash equilibrium as a function of the spreading factor N. A network with K=10 users is considered, while the aPDP is assumed to be exponentially decaying with  $\Lambda=20\,{\rm dB}$ . The number of paths is L=200, thus satisfying the large-system assumption. Red (light) and blue (dark) colors depict the cases ARake (r=1) and PRake (r=0.2), respectively. Lines represent theoretical results provided by (17). In particular, solid lines show analytical values for DS-CDMA  $(N_c=1)$ , while dashed and dotted lines report the IR-UWB scenario, with  $N_c=10$  and  $N_c=50$ , respectively. The markers show the simulation results averaged over  $10\,000$  network realizations. It can be

<sup>&</sup>lt;sup>2</sup>As expected, larger spreading factors better mitigate multipath effects.

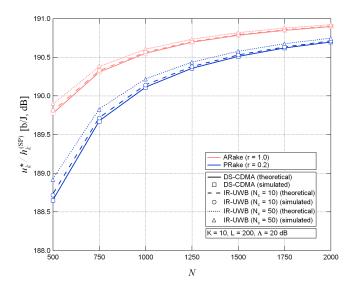


Fig. 2. Comparison of normalized utilities versus processing gain for DS-CDMA and IR-UWB schemes.

seen that the analytical results perfectly match the actual performance of systems. As expected, the performance loss of DS-CDMA wrt IR-UWB is negligible (less than 1 dB) when compared with the normalized achieved utilities. Furthermore, with N fixed, numerical results confirm that a higher r provides smaller difference in performance between the two multiple access schemes. Moreover, such loss decreases as N increases, due to the inherent resistance to multipath, thus smoothing the performance behavior.

Similar considerations can be made when observing the results shown in Fig. 3. The loss of a DS-CDMA wrt an IR-UWB with  $N_c = 50$  is studied. The decay constant of the channel is assumed to be  $\Lambda = 20 \, \mathrm{dB}$ . For the sake of presentation, only analytical results are reported. Red (light) and blue (dark) colors represent K = 10 and K = 20, respectively. The solid lines depict the case ARake, while the dashed lines show the case PRake (r = 0.2). The square markers and the circles report the results with L=200 and L=500 multiple paths, respectively. It can be seen that the loss  $\Phi$  is always very small. In addition, it is worth noting that, for both N and  $N_c$  fixed,  $\Phi$  decreases as L increases. This can be justified since IR-UWB access scheme cannot further mitigate the effects of denser and denser multipath in a  $N_c$ -fixed scenario, and thus its behavior is more similar to that of DS-CDMA systems. This statement is not in contrast with the comments on Fig. 2, since the comparison would be completely different if we considered IR-UWB with fixed N and variable  $N_c$ . In fact, if we choose  $N_c$  such that  $\rho$  is constant accordingly to the increasing L,  $\Phi$  remains unchanged, as is apparent from (21).

# V. CONCLUSION

In this paper, two multiple access schemes, namely DS-CDMA and IR-UWB, have been compared in the context of game-theoretic energy-efficient power control. We have used

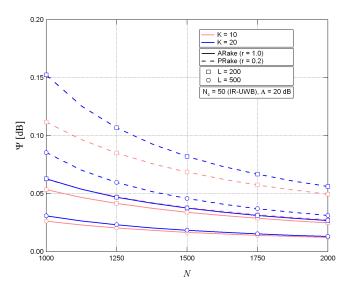


Fig. 3. Performance loss of DS-CDMA wrt IR-UWB for different values of the system parameters.

a large-system analysis to study the performance of a wireless data network using Rake receivers at the access point in a frequency-selective fading channel. Considering systems with equal spreading factor, a measure of the loss of DS-CDMA scheme with respect to IR-UWB multiple access technique has been derived which is dependent only on the network parameters. Theoretical analysis, supported by experimental results, shows that the considered multiple access schemes are practically equivalent in terms of energy efficiency.

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